Instructor: **Grøndahl**

Name: Solutions

MATH 241

Project 2

Unit One Refresher

Due Wednesday November 15th

Be Sure to Use the Techniques from Unit One

1) The graph below shows the total distance s traveled by a bicyclist after t hours.

   a. Estimate the bicyclist's average speed over the time interval $[0,6]$ hours

   $$\frac{s(6)-s(0)}{6-0} = \frac{3.5-0}{6} = \frac{7}{12} \text{ DIST/HOUR}$$

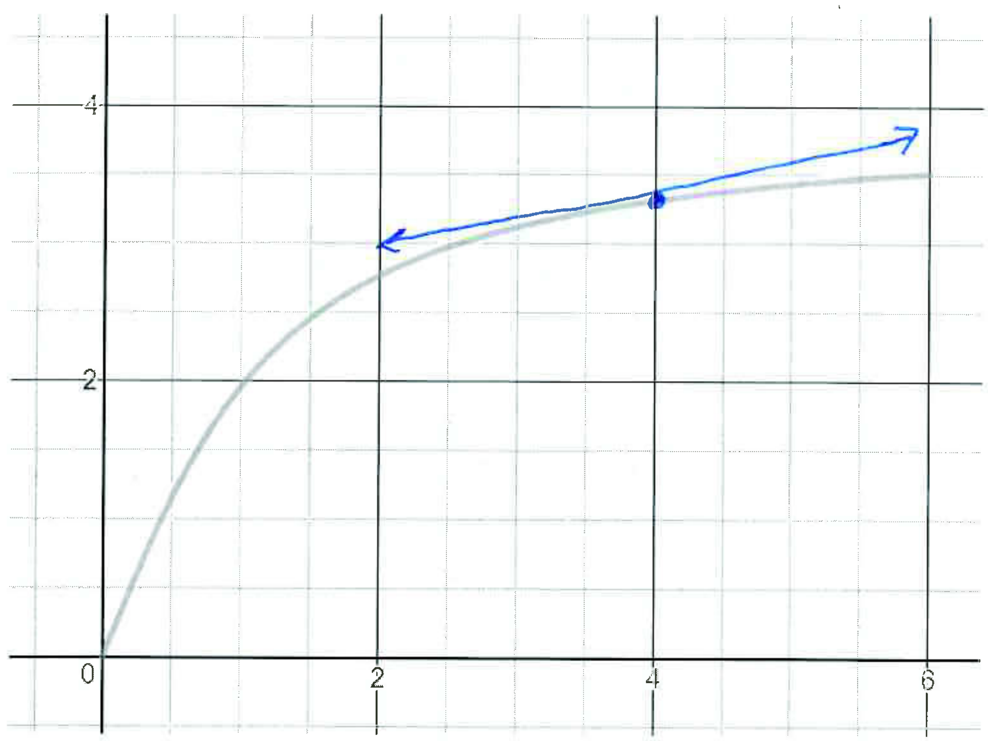
   b. Estimate the bicyclist's instantaneous speed at time $t = 4$ (Help show your work by sketching on the graph)

   $$\frac{3.8-3}{6-2} = \frac{.8}{4} = \frac{1}{5} \text{ DIST/HOUR}$$

   c. Estimate the time at which the cyclist reached maximum speed.

   $t = 0$ HR

   SLOPE IS LARGEST



2) Find the slope of $y = \frac{x}{x+1}$ at $\left(2, \frac{2}{3}\right)$ by determining the value that the average rate of change approaches as $h \to 0$

$$m = \frac{\frac{2+h}{2+h+1} - \frac{2}{2+1}}{h} = \frac{6+3h-6-2h}{h(3+h)(3)} = \frac{h}{h(3+h)(3)}$$

$$= \frac{1}{(3+h)(3)}, \quad h \neq 0$$

So

As $h \to 0$

$$m \to \frac{1}{(3+0)(3)} = \frac{1}{9}$$

3) Evaluate $\lim_{x \to 5} \frac{x^2 - x - 20}{x^2 - 25}$ by simplifying the expression to something determinant.

$$= \lim_{x \to 5} \frac{(x-5)(x+4)}{(x-5)(x+5)}$$

$$= \lim_{x \to 5} \frac{x+4}{x+5}$$

$$= \frac{9}{10}$$

4) Use the appropriate theorems or limit properties to evaluate $\lim_{x\to 0}\dfrac{\sin 5x}{\sin 4x}$

$$= \lim_{x\to 0} \frac{\sin 5x\,(x)}{\sin 4x\,(x)}$$

$$= \left(\lim_{x\to 0} \frac{\sin 5x}{x}\right)\left(\lim_{x\to 0} \frac{x}{\sin 4x}\right)$$

$$= \left(\lim_{x\to 0} \frac{\sin 5x}{x}\right)\left(\frac{1}{\lim_{x\to 0} \frac{\sin 4x}{x}}\right)$$

Let $a = 5x$

$\lim_{x\to 0} a = \lim_{x\to 0} 5x = 0$

Let $b = 4x$

$\lim_{x\to 0} b = \lim_{x\to 0} 4x = 0$

$$= \left(\lim_{a\to 0} \frac{\sin a}{a/5}\right)\left(\frac{1}{\lim_{b\to 0} \frac{\sin b}{b/4}}\right)$$

$$= \frac{5}{4}\left(\lim_{a\to 0} \frac{\sin a}{a}\right)\left(\frac{1}{\lim_{b\to 0} \frac{\sin b}{b}}\right)$$

$$= \frac{5}{4}(1)(1)$$

$$= \frac{5}{4}$$

5) Use the graph below to find the following or state that it doesn't exist. If it doesn't exist, explain why.

a. $f(2)$ $= 2$

b. $\lim_{x\to 2} f(x) = 1$

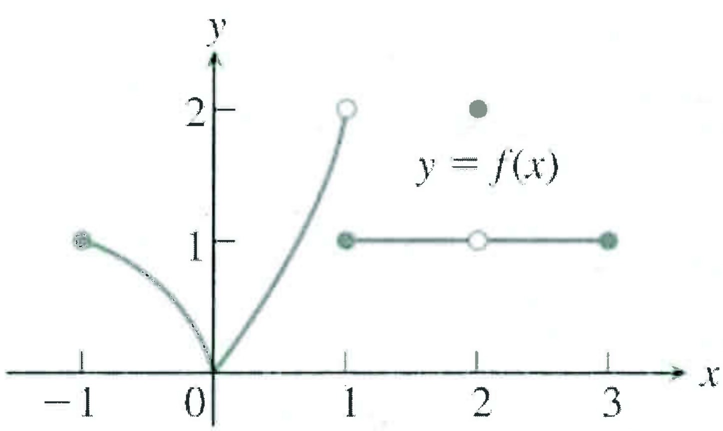
c. $\lim_{x\to 1} f(x)$ DNE $\lim_{x\to 1^-} f(x) = 2 \neq 1 = \lim_{x\to 1^+} f(x)$

d. $\lim_{x\to 1^+} f(x) = 1$

e. $\lim_{x\to 3^-} f(x) = 1$

f. $\lim_{x\to -1} f(x)$ DNE $\lim_{x\to -1^-}$ DNE BECAUSE $f(x)$ IS UND FOR $x < -1$

g. $f(-2)$ DNE DOMAIN $x \geq -1$



6) Give the equation of a continuous function that has 3 discontinuities.

ANSWERS VARIES

$$f(x) = \frac{1}{(x-1)\,x\,(x+1)}$$

7) Use the $\varepsilon-\delta$ definition of limit to prove that $\lim_{x\to 1} x^2 = 1$

GIVEN $\varepsilon>0$, NEED TO SHOW THERE EXIST $\delta>0$

s.t. $0<|x-1|<\delta \Rightarrow |x^2-1|<\varepsilon$

So

$0<|x-1|<\delta$

$-\delta<x-1<\delta$, $x\neq 1$

$1-\delta<x<\delta+1$

$|x^2-1|<\varepsilon$

$-\varepsilon<x^2-1<\varepsilon$

$1-\varepsilon<x^2<\varepsilon+1$

$\sqrt{1-\varepsilon}<x<\sqrt{\varepsilon+1}$

So WANT

$\sqrt{1-\varepsilon}<1-\delta<x<\delta+1<\sqrt{\varepsilon+1}$

$\sqrt{1-\varepsilon}<1-\delta$ AND $\delta+1<\sqrt{\varepsilon+1}$

$\delta<1-\sqrt{1-\varepsilon}$ AND $\delta<\sqrt{\varepsilon+1}-1$

So $\delta=\sqrt{\varepsilon+1}-1$

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8) Use limits to determine what value of a makes this function continuous.

$$f(x)=\begin{cases} x^3-1 & ,x<-2 \\ 4ax & ,x\geq \cancel{2}-2 \end{cases}$$

$(-2)^3-1 = 4a(-2)$

$-8-1 = -8a$

$-9 = -8a$

$a = 9/8$

OR IF YOU USED $f(x)=\begin{cases} x^3-1, & x<2 \\ 4ax, & x\geq 2 \end{cases}$

$(2)^3-1 = 4a(2)$

$7 = 8a$

$a = 7/8$

9) Find $\lim_{x \to -\infty} \frac{4-3x^3}{\sqrt{x^6+9}}$ by simplifying the expression to something determinant.

$$\lim_{x \to -\infty} \frac{(4-3x^3)\,{}^1/_{x^3}}{(\sqrt{x^6+9})\,{}^1/_{x^3}}$$

$$= \lim_{x \to -\infty} \frac{\frac{4}{x^3} - 3}{\sqrt{1 + {}^9/_{x^6}}}$$

$$= \frac{0-3}{\sqrt{1+0}}$$

$$= -3$$