Instructor: **Grøndahl**

Name:____

MATH 241

Project 3

Fall 2017

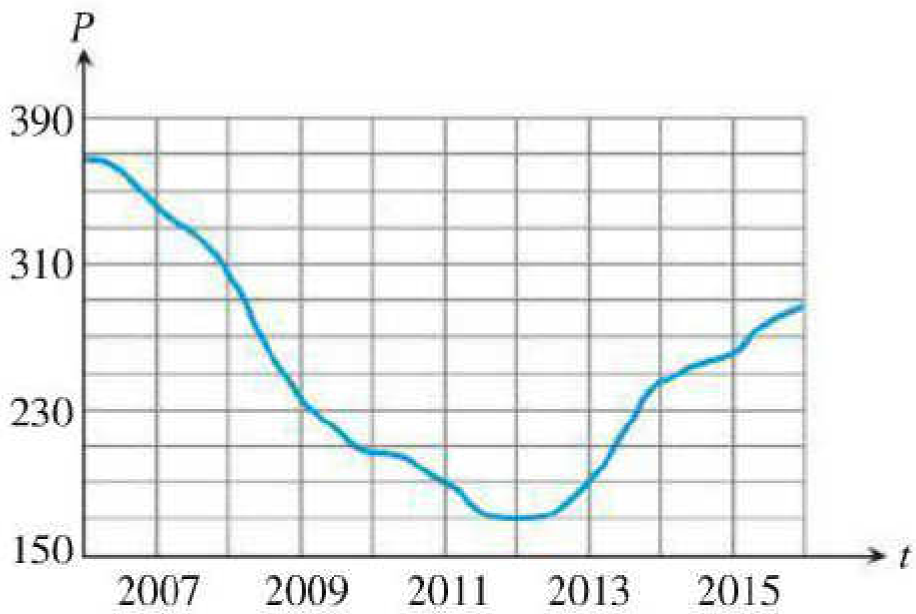
Due Monday November 27th

Show all work

Justify All Conclusions

1) In the late 1860s, Adolf Fick, a professor of physiology in the Faculty of Medicine in Wurzberg, Germany, developed one of the methods we use today for measuring how much blood your heart pumps in a minute. Your cardiac output as you read this sentence is about 7 L/min. At rest it is likely a bit under 6 L/min. If you are a trained marathon runner running a marathon, your cardiac output can be as high as 30 L/min.
   Your cardiac output can be calculated with the formula
   $$y = \frac{Q}{D}$$
   where Q is the number of milliliters of CO_2 you exhale in a minute and D is the difference between the CO_2 concentration in the blood returning to the lungs.
   Suppose $Q = 233$ ml/min and $D = 97 - 56 = 41$ ml/L and we know that D is decreasing at a rate of 2 ml/L per minute, but Q remains unchanged. What is happening to the cardiac output?

2) Average single-family home prices P (in thousands of dollars) in Sacramento, California, are shown in the accompanying figure from 2006 to 2015



a. Estimate the rate of change of home prices in 2011

b. In what year are the home prices increasing most rapidly?

c. Estimate the rate of change at that time.

d. In what year are the home prices decreasing most rapidly?

e. Estimate the rate of change at that time.

3) Suppose that the dollar cost of producing x washing machines is $c(x) = 2000 + 100x - 0.1x^2$

   a. Find the average cost per machine of producing the first 100 machines.

   b. Find the marginal cost when 100 washing machines are produced.

   c. Show that the marginal cost when 100 washing machines are produced is approximately the cost of producing one more washing machine after the first 100 have been made.

4) Is there a value of b that will make $f(x)=\begin{cases} x+b & ,x<0 \\ \cos x & ,x\geq 0 \end{cases}$ differentiable at $x=0$? Give reasons for your answer.

5) For oscillations of small amplitude (short swings), we may safely model the relationship between the period T and the length L of a simple pendulum with the equation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g is the constant acceleration due to gravity at the pendulum's location. If the pendulum is made of metal, its length will vary with the temperature u as

$$\frac{dL}{du} = kL$$

where k is a proportionality constant determined by the particular metal and geometry of the pendulum.

Show that the rate of change of period with respect to temperature is $\frac{kT}{2}$

6) Find $\frac{dy}{dx}$ for $x^y = y^x$ assuming $x, y > 0$

7) Find the linearization of $f(x) = (1+x)^k$ at $a = 0$ and then use that linearization to approximate $\sqrt[4]{0.99}$