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Name: Solutions

MATH 241

Project 3

Fall 2017

Due Monday November 27th

Show all work

Justify All Conclusions

1) In the late 1860s, Adolf Fick, a professor of physiology in the Faculty of Medicine in Wurzberg, Germany, developed one of the methods we use today for measuring how much blood your heart pumps in a minute. Your cardiac output as you read this sentence is about 7 L/min. At rest it is likely a bit under 6 L/min. If you are a trained marathon runner running a marathon, your cardiac output can be as high as 30 L/min.
Your cardiac output can be calculated with the formula

$$y = \frac{Q}{D}$$

where Q is the number of milliliters of CO_2 you exhale in a minute and D is the difference between the CO_2 concentration in the blood returning to the lungs.
Suppose $Q = 233$ ml/min and $D = 97 - 56 = 41$ ml/L and we know that D is decreasing at a rate of 2 ml/L per minute, but Q remains unchanged. What is happening to the cardiac output?

$Q = 233$

$D = 41$

$\frac{dQ}{dt} = 0$

$\frac{dD}{dt} = -2$

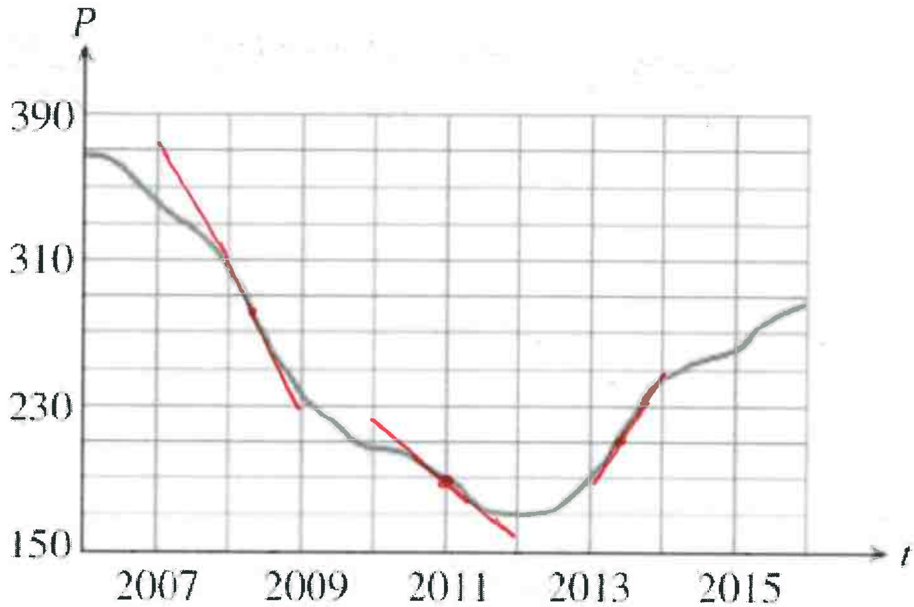
$$\frac{dy}{dt} = \frac{\frac{dQ}{dt} D - Q \frac{dD}{dt}}{D^2}$$

$$= \frac{0(41) - (233)(-2)}{41^2}$$

$$\approx 0.277 \text{ L/min}^2$$

CARDIC OUTPUT IS INCREASING.

2) Average single-family home prices P (in thousands of dollars) in Sacramento, California, are shown in the accompanying figure from 2006 to 2015



a. Estimate the rate of change of home prices in 2011

$\frac{160-220}{2012-2010} = \frac{-60}{2} = -30$ THOUSAND DOLLARS / YEAR

$= -\$2{,}500$ / MONTH

b. In what year are the home prices increasing most rapidly?

2013

c. Estimate the rate of change at that time.

$\frac{250-190}{2014-2013} = \frac{60}{1} =$ 60 THOUSAND DOLLAR / YEAR

$= \$5{,}000$ / MONTH

d. In what year are the home prices decreasing most rapidly?

2008

e. Estimate the rate of change at that time.

$\frac{230-370}{2009-2007} = \frac{-140}{2} = -70$ THOUSAND DOLLARS / YEAR

$= -\$5{,}833$ / MONTH

3) Suppose that the dollar cost of producing x washing machines is $c(x) = 2000 + 100x - 0.1x^2$

a. Find the average cost per machine of producing the first 100 machines.

$$\frac{C(100)}{100} = \frac{2000 + 10000 - 1000}{100} = \frac{11000}{100} = \$110$$

b. Find the marginal cost when 100 washing machines are produced.

$$C'(x) = 100 - 0.2x$$
$$C'(100) = 100 - 0.2(100)$$
$$= 100 - 20$$
$$= \$80$$

c. Show that the marginal cost when 100 washing machines are produced is approximately the cost of producing one more washing machine after the first 100 have been made.

$$C(101) - C(100) = (2000 + 100(101) - 0.1(101)^2) - (11000)$$
$$= (2000 + 10100 - 1020.1) - 11000$$
$$= 11{,}079.9 - 11{,}000$$
$$= \$79.90 \approx \$80$$

4) Is there a value of b that will make $f(x)=\begin{cases} x+b & ,x<0 \\ \cos x & ,x\geq 0 \end{cases}$ differentiable at $x=0$? Give reasons for your answer.

APPROACHING FROM LEFT $(x<0)$

$f'(x) = 1$

$f'(0) = 1$

APPROACHING FROM RIGHT $(x\geq 0)$

$f'(x) = -\sin x$

$f'(0) = 0$

SINCE b HAS NO EFFECT ON $f'(x)$

WE CAN'T MAKE $1=0$

5) For oscillations of small amplitude (short swings), we may safely model the relationship between the period T and the length L of a simple pendulum with the equation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g is the constant acceleration due to gravity at the pendulum's location. If the pendulum is made of metal, its length will vary with the temperature u as

$$\frac{dL}{du} = kL$$

where k is a proportionality constant determined by the particular metal and geometry of the pendulum.

Show that the rate of change of period with respect to temperature is $\frac{kT}{2}$

$$\frac{dT}{du} = 2\pi\left(\frac{1}{2}\right)\left(\frac{L}{g}\right)^{-1/2}\left(\frac{dL}{du}\right)\left(\frac{1}{g}\right)$$

$$= \pi\sqrt{\frac{g}{L}}\,kL\left(\frac{1}{g}\right)$$

$$= \pi k\sqrt{\frac{L}{g}}$$

$$= \frac{kT}{2}$$

6) Find $\frac{dy}{dx}$ for $x^y = y^x$ assuming $x, y > 0$

$$\ln x^y = \ln y^x$$

$$y \ln x = x \ln y$$

$$\frac{d}{dx}(y \ln x) = \frac{d}{dx}(x \ln y)$$

$$\frac{dy}{dx}\ln x + \frac{y}{x} = (1)\ln y + x\frac{1}{y}\frac{dy}{dx}$$

$$\frac{dy}{dx}\ln x - \frac{x}{y}\left(\frac{dy}{dx}\right) = \ln y - \frac{y}{x}$$

$$\frac{dy}{dx}\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$$

7) Find the linearization of $f(x)=(1+x)^k$ at $a=0$ and then use that linearization to approximate $\sqrt[4]{0.99}$

$$f(0) = (1+0)^k = 1$$

$$f'(x) = k(1+x)^{k-1}$$

$$f'(0) = k(1)^{k-1} = k$$

$$L(x) = 1 + kx$$

$$\sqrt[4]{0.99} = (1-0.01)^{1/4}$$

$$\approx 1 + \tfrac{1}{4}(-0.01)$$

$$= 1 - .0025$$

$$= 0.9975$$