Instructor: **Grøndahl**

Name: KEY

MATH 241

Unit 1 Exam

Fall 2017

No Calculators or Notes Allowed

Show all work

Justify All Conclusions

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1) The graph below shows the total distance s traveled by a bicyclist after t hours.

   a. Estimate the bicyclist's average speed over the time interval $[1, 3.5]$ hours

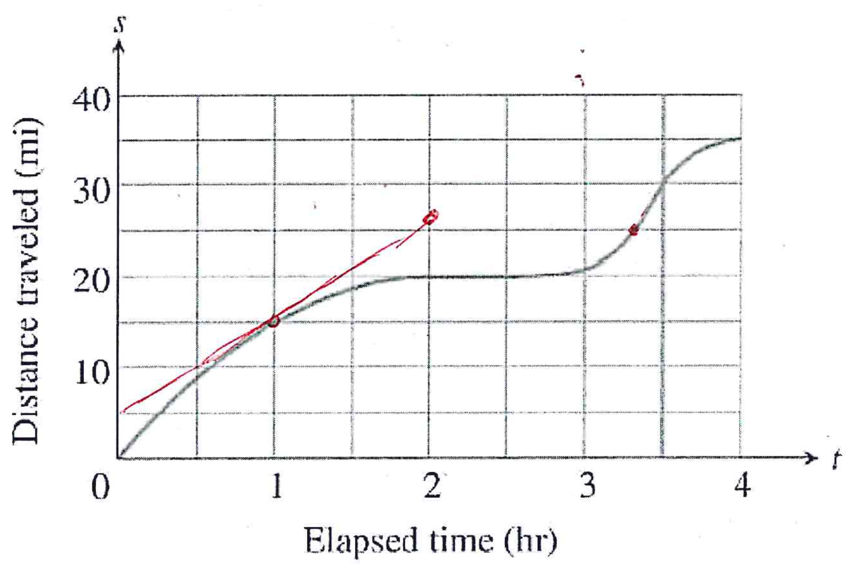
   $$\frac{\Delta s}{\Delta t} = \frac{30-15}{3.5-1} = \frac{15}{2.5} = 6 \text{ mPH}$$

   b. Estimate the bicyclist's instantaneous speed at time $t = 1$

   $$\frac{ds}{dt} = \frac{27-15}{2-1} = 12 \text{ mPH}$$

   c. Estimate the time at which the cyclist reached maximum speed.

   $t = 3.3$ HRS



2) Find the slope of $y=\frac{1}{x}$ at $\left(2,\frac{1}{2}\right)$ by determining the value that the average rate of change approaches as $h\to 0$

$$\frac{\Delta y}{\Delta x}=\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{\frac{x-x-h}{x(x+h)}}{h}$$

$$=\frac{-h}{xh(x+h)}$$

$$=\frac{-1}{x(x+h)}\ ,\ h\neq 0$$

$$=\frac{-1}{2(2+h)}\ ,\ \text{For } x=2$$

AND $\frac{-1}{2(2+h)}\to\frac{-1}{2(2)}=\frac{-1}{4}$ AS $h\to 0$

3) Evaluate $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}$ by simplifying the expression to something determinant.

$$= \lim_{x\to 9} \frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{6}$$

4) Use the appropriate theorems or limit properties to evaluate $\lim_{x\to\infty} \frac{\cos x}{3x}$

$$\frac{-1}{3x} \le \frac{\cos x}{3x} \le \frac{1}{3x}$$

$$\lim_{x\to\infty} \frac{-1}{3x} = 0$$

$$\lim_{x\to\infty} \frac{1}{3x} = 0$$

So $\lim_{x\to\infty} \frac{\cos x}{3x} = 0$

By SANDWICH THM.

5) Use the graph below to find the following or state that it doesn't exist

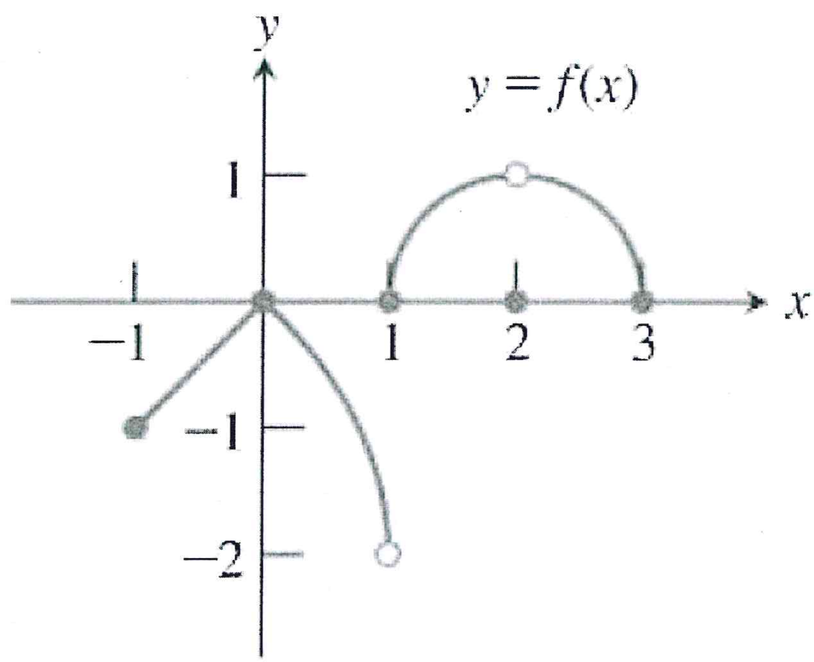
a. $f(2)$ $= 0$

b. $\lim_{x \to 2} f(x)$ $= 1$

c. $\lim_{x \to 1} f(x)$ DNE

d. $\lim_{x \to -1} f(x)$ DNE

e. $f(-2)$ DNE



6) Is it possible for a continuous function to have discontinuities? Explain.

YES. A CONTINUOUS FUNCTION NEED ONLY BE CONTINUOUS ON ITS DOMAIN WHILE DISCONTINUITIES CAN OCCUR AT POINTS NOT IN THE DOMAIN.

7) Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x\to 4}\sqrt{x} = 2$

GIVEN $\varepsilon > 0$ WE WANT TO FIND $\delta > 0$

SUCH THAT

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

IN THIS CASE

$$0 < |x - 4| < \delta \Rightarrow |\sqrt{x} - 2| < \varepsilon$$

$$-\delta < x - 4 < \delta,\ x \neq 4 \Rightarrow -\varepsilon < \sqrt{x} - 2 < \varepsilon$$

$$4 - \delta < x < 4 + \delta,\ x \neq 4 \Rightarrow 2 - \varepsilon < \sqrt{x} < 2 + \varepsilon$$

$$(2-\varepsilon)^2 < x < (2+\varepsilon)^2$$

SO WE NEED

$$(2-\varepsilon)^2 < 4 - \delta < x < 4 + \delta < (2+\varepsilon)^2$$

SO

$$(2-\varepsilon)^2 < 4 - \delta \quad \text{AND} \quad 4 + \delta < (2+\varepsilon)^2$$

$$\delta < 4 - (2-\varepsilon)^2 \quad \text{AND} \quad \delta < (2+\varepsilon)^2 - 4$$

$$\delta < 4 - (4 - 4\varepsilon + \varepsilon^2) \quad \text{AND} \quad \delta < 4 + 4\varepsilon + \varepsilon^2 - 4$$

$$\delta < 4\varepsilon - \varepsilon^2 \quad \text{AND} \quad \delta < 4\varepsilon + \varepsilon^2$$

SO $\quad \delta < 4\varepsilon - \varepsilon^2$

8) For what value of a is the function $f(x)=\begin{cases} x^2-1 & ,x<3 \\ 2ax & ,x\geq 3 \end{cases}$ continuous.

WE WANT THE TWO PIECES
TO CONNECT AT X=3

So

$$(3)^2-1 = 2a(3)$$

$$8 = 6a$$

$$\frac{4}{3} = a$$

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9) Find $\lim_{x\to\infty}\frac{(9x^4+x)\,1/x^4}{(-x^4+5x^2+2)\,1/x^4}$

$$= \lim_{x\to\infty} \frac{9+\frac{1}{x^3}}{-1+\frac{5}{x^2}+\frac{2}{x^4}}$$

$$= \frac{9+0}{-1+0+0}$$

$$= -9$$