Instructor: **Grøndahl**

Name: Solutions

MATH 241

Unit 2 Exam

Fall 2017

No Calculators or Notes Allowed

Show all work

Justify All Conclusions

1) Does the graph of $f(x)=\begin{cases}-1 & ,x<0\\ 0 & ,x=0\\ 1 & ,x>0\end{cases}$ have a vertical tangent at the origin?

Justify your answer using one-sided limits.

$$\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-}\frac{-1-0}{h} = \lim_{h\to 0^-}\frac{-1}{h} = \infty$$

$$\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+}\frac{1-0}{h} = \infty$$

So $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \infty$

So YES.

OR ——

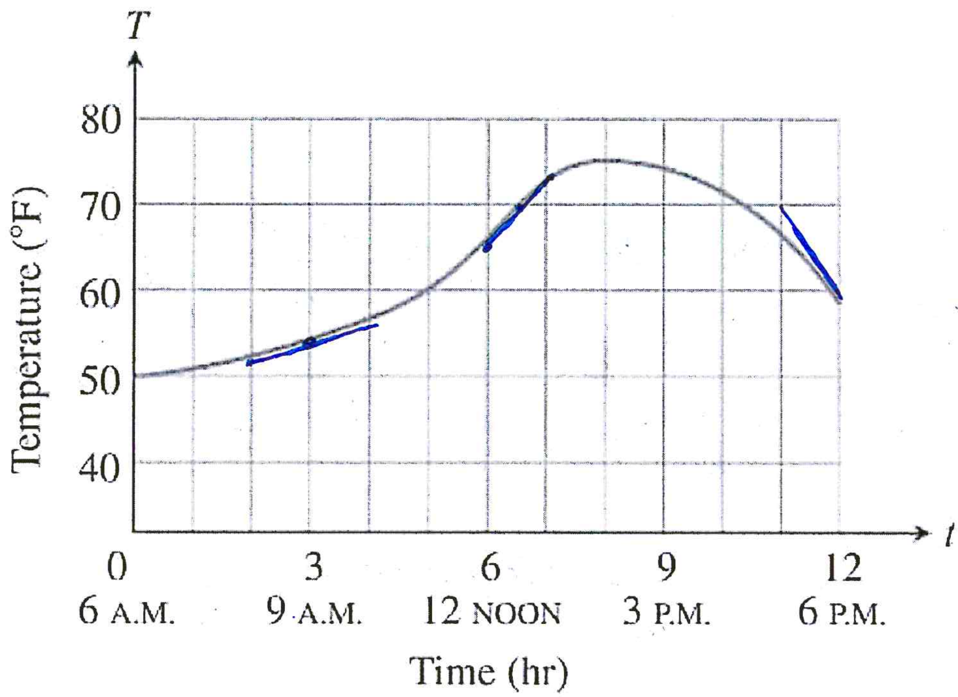
SINCE THE TEXT IMPLIED THAT THE FTN. NEEDS TO BE CTN. TO HAVE A VERTICAL TANGENT AND

$$\lim_{x\to 0^-} f(x) = -1 \neq \lim_{x\to 0^+} f(x) = 1$$

SO $f(x)$ IS NOT CTN AT 0

SO NO

2) The given graph shows the temperature T in $^\circ F$ at Davis, CA on April 18, 2008 between 6 a.m. and 6 p.m.



a. Estimate the rate of change in the temperature at 9 a.m.

$$\approx \frac{56-51}{4-2} = \frac{5}{2} = 2.5\ ^\circ F/HR$$

b. At what time is the temperature increasing most rapidly?

$\approx$ 12:30PM

SLOPE IS LARGEST

c. Estimate the rate of temperature change at that time.

$$\approx \frac{74-65}{7-6} = \frac{9}{1} = 9^\circ F/HR$$

d. At what time is the temperature decreasing most rapidly?

$\approx$ 6pm

e. Estimate the rate of temperature change at that time.

$$\approx \frac{60-70}{12-11} = -10^\circ F/HR$$

3) Based on the data from the U.S. Bureau of Public Records, a model for the total stopping distance of a moving car in terms of its speed is,

$$s(v) = 1.1v + 0.054v^2$$

where s is measured in feet and v in miles per hour. The linear term models the distance the car travels after the driver perceives a need to stop and before the driver applies the brake. The quadratic term models the distance the car travels with the brakes applied.

Find $\left.\frac{ds}{dv}\right|_{v=70}$ and interpret its meaning.

$$\frac{ds}{dv} = 1.1 + 0.108v$$

$$\left.\frac{ds}{dv}\right|_{v=70} = 1.1 + 0.108(70)$$

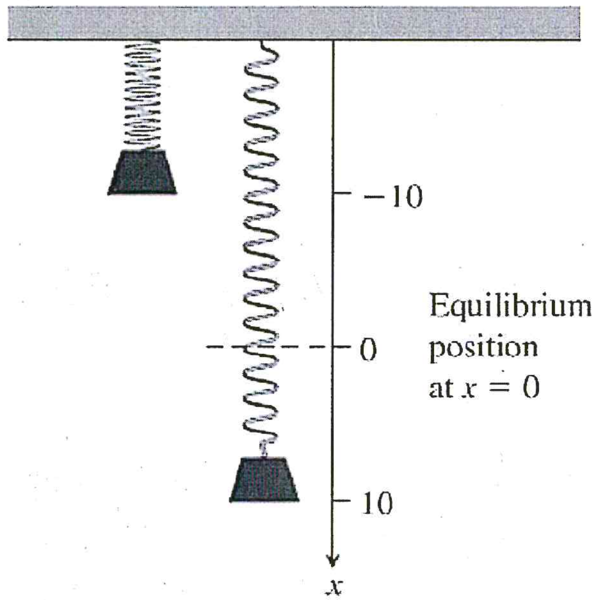
$$= 8.66 \text{ FT/MPH}$$

AT 70 MPH THE STOPPING DISTANCE IS INCREASING AT 8.66 FT PER MPH

4) A weight is attached to a spring and reaches its equilibrium position $(x=0)$. It is then set in motion resulting in a displacement of

$$x(t) = 10\cos t$$

where x is measured in centimeters and t is measured in seconds.



Find the spring's velocity when $t = \frac{\pi}{3}$

$$v(t) = \frac{dx}{dt} = -10\sin t$$

$$v\left(\frac{\pi}{3}\right) = -10\left(\frac{\sqrt{3}}{2}\right)$$

$$= -5\sqrt{3} \text{ cm/s}$$

Is the weight moving up or down at this time?

MOVING UP

(NEGATIVE X-DIR)

5) For oscillations of small amplitude (short swings), we may safely model the relationship between the period T and the length L of a simple pendulum with the equation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g is the constant acceleration due to gravity at the pendulum's location. If the pendulum is made of metal, its length will vary with the temperature u as

$$\frac{dL}{du} = kL$$

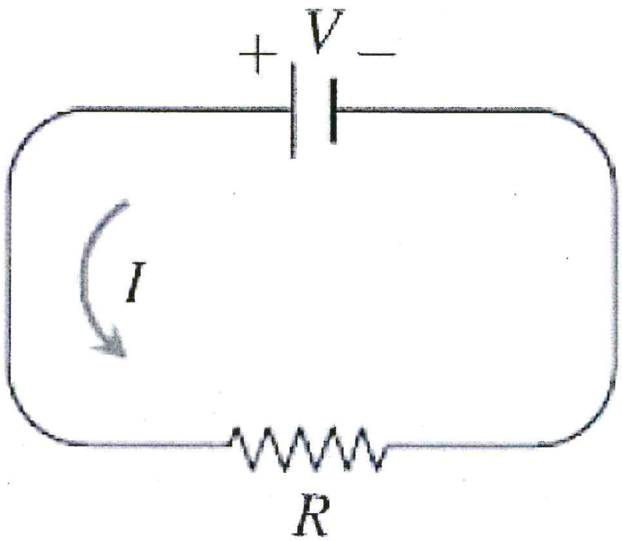
where k is a proportionality constant determined by the particular metal and geometry of the pendulum.

Show that the rate of change of period with respect to temperature is $\dfrac{kT}{2}$

$$\frac{dT}{du} = 2\pi\left(\frac{1}{2}\right)\left(\frac{L}{g}\right)^{-1/2}\left(\frac{1}{g}\right)\frac{dL}{du}$$

$$= \pi\sqrt{\frac{g}{L}}\,\frac{kL}{g} = \frac{kT}{2}$$

6) The voltage V (volts), current I (amperes), and resistance R (ohms) of an electric circuit like the one shown here are related by the equation $V = IR$. Suppose that V is increasing at a rate of 1 volt/sec. while I is decreasing at a rate of $\frac{1}{3}$ amp/sec. Find the rate at which R is changing when $V = 12$ volts and $I = 2$ amps.



$V = IR$

$12 = 2R$

$R = 6$ ohms

$$\frac{dV}{dt} = \frac{dI}{dt} R + I \frac{dR}{dt}$$

$$1 = (-1/3)6 + 2 \frac{dR}{dt}$$

$$\frac{dR}{dt} = 3/2 \text{ ohm/sec}$$

7) Find the linearization of $f(x) = (1+x)^k$ at $a = 0$ and then use that linearization to approximate $\sqrt[3]{1.009}$

$f(0) = (1+0)^k = 1$

$f'(x) = k(1+x)^{k-1}$

$f'(0) = k(1+0)^{k-1} = k$

$L(x) = 1 + k(x-0)$

$= 1 + kx$

$\sqrt[3]{1.009} \approx L(.009) = 1 + \left(\frac{1}{3}\right)\left(\frac{9}{1000}\right)$

$= 1.003$