Instructor: **Grøndahl**

Name: Solutions

MATH 241

Unit 4 Exam

Fall 2017

No Calculators or Notes Allowed

Show all work

Justify All Conclusions

1) An object is dropped straight down from a helicopter. The object falls faster and faster but its acceleration decreases over time because of air resistance. The acceleration is measured in $\mathrm{ft/sec^2}$ and recorded every second after the drop for 5 seconds. Use the data below to find a lower estimate for the speed at time $t = 5$

t	0	1	2	3	4	5
a	32.00	19.41	11.77	7.14	4.33	2.63

SPEED $> (1)(19.41 + 11.77 + 7.14 + 4.33 + 2.63)$

$= 45.28$ Ft/s

2) Write the sum $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$ in sigma notation

$$\sum_{k=1}^{5} \frac{(-1)^{k+1}}{k}$$

3) Evaluate the sum $\sum_{k=1}^{50}\left[(k+1)^2-k^2\right]$

$$= \sum_{k=1}^{50} k^2+2k+1-k^2$$

$$= 2\sum_{k=1}^{50} k + \sum_{k=1}^{50} 1$$

$$= 2\,\frac{50(51)}{2} + (1)50$$

$$= 2550 + 50$$

$$= 2600$$

4) Find a formula for the Riemann sum of $f(x) = x + x^2$ obtained by dividing the interval $[0,1]$ into n equal subintervals and using the right-hand endpoint for c_k. Then take the limit as $n \to \infty$ to calculate the area under the curve.

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$f(c_k) = f(0+k\Delta x) = f\left(\frac{k}{n}\right) = \frac{k}{n} + \frac{k^2}{n^2}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n}\left(\frac{k}{n}+\frac{k^2}{n^2}\right)\frac{1}{n}$$

$$= \lim_{n\to\infty}\left(\frac{1}{n^2}\sum_{k=1}^{n} k + \frac{1}{n^3}\sum_{k=1}^{n} k^2\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right)$$

$$= \lim_{n\to\infty} \frac{1}{2}(1)\left(1+\frac{1}{n}\right) + \frac{1}{6}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$= \frac{1}{2}(1)(1) + \frac{1}{6}(1)(1)(2)$$

$$= \frac{1}{2} + \frac{1}{3}$$

$$= 5/6$$

5) Express $\lim\limits_{\|P\|\to 0}\sum\limits_{k=1}^{n}\dfrac{1}{1-c_k}\Delta x_k$ where P is the partition of $[2,3]$ as a definite integral.

$$= \int_2^3 \frac{1}{1-x}\,dx$$

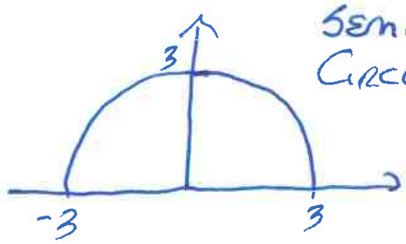
6) Assuming $\int\limits_1^2 f(x)dx=-4$, $\int\limits_1^5 f(x)dx=6$, and $\int\limits_1^5 g(x)dx=8$ find $\int\limits_2^5 f(x)dx$

$$\int_1^2 f(x)\,dx = \int_1^5 f(x)\,dx - \int_2^5 f(x)\,dx$$

$$-4 = 6 - \int_2^5 f(x)\,dx$$

$$\int_2^5 f(x)\,dx = 10$$

7) Use known area to find $\int\limits_{-3}^{3}\sqrt{9-x^2}\,dx$



Semi-Circle

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$$

8) Find $\frac{dy}{dx}$ for $y = \int_{\sqrt{x}}^{0} \sin\left(t^2\right) dt$

$$y = -\int_{0}^{\sqrt{x}} \sin(t^2)\, dt$$

$$\frac{dy}{dx} = -\sin\left(\left(\sqrt{x}\right)^2\right)\frac{1}{2\sqrt{x}} = \frac{-\sin(x)}{2\sqrt{x}}$$

9) Evaluate $\int_{\pi/4}^{3\pi/4} \csc\theta \cot\theta \, d\theta$

$$= \Big[-\csc\theta\Big]_{\pi/4}^{3\pi/4}$$

$$= -\Big[\csc\theta\Big]_{\pi/4}^{3\pi/4}$$

$$= -\left(-\sqrt{2} - \sqrt{2}\right)$$

$$= 2\sqrt{2}$$

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10) Evaluate $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2du = \frac{1}{\sqrt{x}}dx$

$= 2\int \frac{du}{(1+u)^2}$

$v = 1+u$

$dv = du$

$= 2\int \frac{1}{v^2}dv = \frac{-2}{v} + C = \frac{-2}{1+u} + C = \frac{-2}{1+\sqrt{x}} + C$

11) Evaluate $\int_0^{\pi/4} \left(1+e^{\tan\theta}\right)\sec^2\theta\, d\theta$

$u = \tan\theta$

$du = \sec^2\theta\, d\theta$

$= \int_0^1 1+e^u\, du$

$= \left[u+e^u\right]_0^1$

$= (1+e^1) - (0+1)$

$= e$

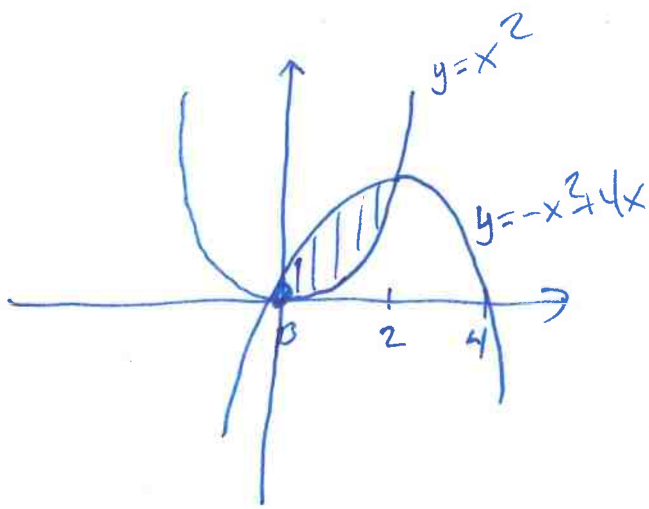
12) Find the area enclosed by $y = x^2$ and $y = -x^2 + 4x$

$$x^2 = -x^2 + 4x$$

$$2x^2 - 4x = 0$$

$$2x(x-2) = 0$$

$$x = 0, 2$$



$$A = \int_0^2 (-x^2 + 4x - x^2)\,dx$$

$$= \int_0^2 (-2x^2 + 4x)\,dx$$

$$= \left[\frac{-2}{3}x^3 + 2x^2 \right]_0^2$$

$$= \left(\frac{-16}{3} + 8\right) - (0 + 0)$$

$$= \frac{8}{3}$$