MATH 242
Final Review

I. Integration Techniques

Be able to use the proper integration technique given an arbitrary integral

U-Substitution	Integration by Parts
Completing the Square	Partial Fractions
Trigonometric Identities	Products of Powers of Sine and Cosine
Improper Fractions	Powers of Tangent and Secant
Separating Fractions	Products of Sines and Cosines
Multiplying by a Form of 1	Trigonometric Substitution
Eliminating Square Roots	

Evaluate the following

1) $\int_0^1 \frac{1}{\sqrt{a}+1}\,da$

2) $\int \sin^2\alpha\cos^2\alpha\; d\alpha$

3) $\int_{\frac{3\pi}{4}}^{\pi} \sqrt{1+\tan^2\beta}\; d\beta$

4) $\int \frac{b^2}{b^2+1}\,db$

5) $\int \frac{c}{\sqrt{c^2+16}}\,dc$

6) $\int \cot^2\delta\; d\delta$

7) $\int \cos^3\phi\; d\phi$

8) $\int g^3e^{-g}\; dg$

9) $\int_0^1 \frac{e^h}{1+e^{2h}}\,dh$

10) $\int \frac{k+17}{k^2+4k-5}\,dk$

11) $\int \cos 2\varphi\cos 4\varphi\; d\varphi$

12) $\int \sec^3\gamma\; d\gamma$

13) $\int \sec(7\lambda)\,d\lambda$

14) $\int e^{2n}\sin 3n\; dn$

15) $\int \frac{p^2+2}{3p^2}\,dp$

16) $\int \sec^4\mu\; d\mu$

17) $\int q2^{q^2}\,dq$

18) $\int_1^{3/2} \frac{1}{\sqrt{2r-r^2}}\,dr$

19) $\int \frac{1+s}{1+s^2}\,ds$

20) $\int \frac{\sqrt{9-t^2}}{t^2}\; dt$

21) $\int \tan^3\nu\; d\nu$

22) $\int_{\pi/2}^{3\pi/2} \sqrt{1-\cos\theta}\; d\theta$

23) $\int \frac{1+\cos\rho}{\sin\rho}\,d\rho$

24) $\int \frac{\ln v}{v}\,dv$

25) $\int \sin^3\tau\cos^7\tau\; d\tau$

26) $\int \frac{w^2dw}{\left(w^2-1\right)^{5/2}}$, $w>1$

27) $\int \frac{1}{\csc\omega+\cot\omega}\,d\omega$

28) $\int \frac{1}{x^2-4x+9}\,dx$

29) $\int \frac{5y-9}{y^2-9}\,dy$

30) $\int \frac{\sin\psi}{\sqrt{\cos\psi}}\,d\psi$

31) $\int \frac{z^2}{z-1}\,dz$

32) $\int_0^{\pi/3} \tan(\theta)\ln(\cos(\theta))\; d\theta$

33) $\int t^6\cos 2t\; dt$

34) $\int \frac{7}{\left(m^2+9\right)^3}\,dm$

35) Before using the method of partial fractions, what must be true of the rational expression?

36) When evaluating the indefinite integral $\int 3\sqrt{2x-1}dx$, you may want to use the substitution $u=2x-1$, to get that $\int 3\sqrt{2x-1}dx = \int \frac{3}{2}\sqrt{u}du$. Explain why the following related statement about the definite integral would ***not*** be valid. $\int_1^5 3\sqrt{2x-1}dx = \int_1^5 \frac{3}{2}\sqrt{u}du$.

37). Name the two trig identities which will make this integral "do-able": $\int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x+\cos^2 x}\,dx$

II. Improper Integrals and L'Hoptial's Rule

Be able to evaluate improper integrals

Be able to determine whether an improper integral is convergent or divergent by use of Direct or Limit Comparison

38) For this problem, you will find the flaw in the logic.

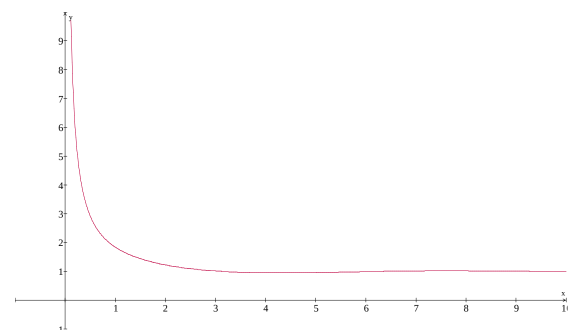
$$\lim_{x\to\infty}\frac{x^2+\sin x}{x^2}=\lim_{x\to\infty}\frac{2x+\cos x}{2x}$$ (both indeterminate forms of type $\frac{\infty}{\infty}$)

$$=\lim_{x\to\infty}\frac{2-\sin x}{2}$$ (which does not exist!)

However, when you graph $f(x)=\dfrac{x^2+\sin x}{x^2}$ you get this:

This graph would seem indicate a limit as $x\to\infty$ of 1.

Explain the seeming contradiction.



III. Sequences

Be able to determine if a sequence converges or diverges and if it converges determine its limit

Be able to find a formula for a given sequence.

39) Each of these sequences converges. Name the theorem(s) and/or definition(s) that may be used to justify this convergence.

(a) $a_n=\dfrac{cos(n)}{n}$ (b) $a_n=cos\left(\dfrac{1}{n}\right)$ (c) $a_n=ne^{-n}$ (d) $a_n=\sqrt[n]{5n^2}$

40) Describe, in your own words, the difference between $\lim_{n\to\infty}a_n=8$ and $\sum_{n=1}^{\infty}a_n=8$.

IV. Series Convergence or Divergence

Be able to determine whether a series converges (conditionally or absolutely) or diverges using the proper theorems and tests

	Definition of Convergence
Geometric Series	Telescoping Series
Nth Term Test	Integral Test
Direct Comparison Test	Limit Comparison Test
Ratio Test	Root Test
Alternating Series Test	Absolute Convergence Test

41) What conditions must be met in order for the DCT for series, DCT for integrals, LCT, Ratio Test, alt series test, etc to be used.

42) Think of *S* as the exact value of the sum of a series and S_n as an approximation to the value of *S*. Briefly describe the meaning of $\left|S - S_n\right|$, use the word error.

43) Under what circumstances would you use the Alternating Series Estimation Theorem and the Remainder Estimation Theorem?

44) Why does the nth term test for divergence not work for convergence as well?

45) Say that the series $\sum_{n=1}^{\infty}(-1)^{n+1}u_n$ satisfies the conditions of the Alternating Series Test. If we use the approximation $\sum_{n=1}^{\infty}(-1)^{n+1}u_n \approx S_{10}$, will R_{10} be positive or negative?

Determine whether the following series converge conditionally, absolutely or diverge and state the test(s) you use.

46) $\sum_{n=1}^{\infty}\frac{1}{n+1}$

47) $\sum_{n=1}^{\infty}(-1)^n\frac{4^n}{n!}$

48) $\sum_{n=0}^{\infty}\frac{n10^n}{n!}$

49) $\sum_{n=1}^{\infty}e^n$

50) $\sum_{n=5}^{\infty}\frac{\ln n}{n}$

51) $\sum_{n=2}^{\infty}\left[\ln(n)-\ln(n+1)\right]$

52) $\sum_{n=3}^{\infty}(-1)^n\frac{n}{n+1}$

53) $\sum_{n=6}^{\infty}\frac{(n!)^2}{(3n)!}$

54) $\sum_{n=4}^{\infty}\frac{n}{\ln n}$

55) $\sum_{n=10}^{\infty}(-1)^n\frac{1}{\sqrt{n+1}}$

56) $\sum_{n=5}^{\infty}\frac{\tan^{-1}n}{1+n^2}$

57) $\sum_{n=7}^{\infty}\frac{4n^2-6n+3}{3n^4+8n-2}$

58) $\sum_{n=8}^{\infty}\frac{1+5^n}{3^n}$

59) $\sum_{n=15}^{\infty}\frac{n^2}{3^n}$

60) $\sum_{n=0}^{\infty}\left(\frac{2}{n+4}-\frac{2}{n+5}\right)$

61) $\sum_{n=10}^{\infty}\frac{(-1)^n\sin^2 n}{n^2}$

62) $\sum_{n=1}^{\infty}e^{-n}$

63) $\sum_{n=2}^{\infty}\tan\left(\frac{1}{n}\right)$

64) $\sum_{n=3}^{\infty}\frac{6+4^n}{7^n}$

65) $\sum_{n=4}^{\infty}\frac{1}{n^2+1}$

66) $\sum_{n=5}^{\infty}\frac{n^n}{(2n+1)^n}$

67) $\sum_{n=6}^{\infty}\frac{(-1)^{n+1}n^2}{2n+3}$

68) $\sum_{n=7}^{\infty}\frac{1}{n(1+\ln n)}$

69) $\sum_{n=1}^{\infty}\frac{3(n-1)!}{n^3}$

70) $\sum_{n=9}^{\infty}(-1)^{n+2}\frac{\ln n}{n}$

71) $\sum_{n=11}^{\infty}\frac{(\ln n)^3}{n^4}$

72) $\sum_{n=912}^{\infty}\frac{n^2+2n+3}{4n+6}$

V. Other Series Topics

Be able to find the interval and radius of convergence as well as points of conditional convergence for a power series.
Be able to find Taylor series and polynomials
Be able to establish an error bound when using a Taylor polynomial approximation

73) Find the Maclaurin series for $f(x) = xe^{x}$. Integrate over [0, 1] to show that

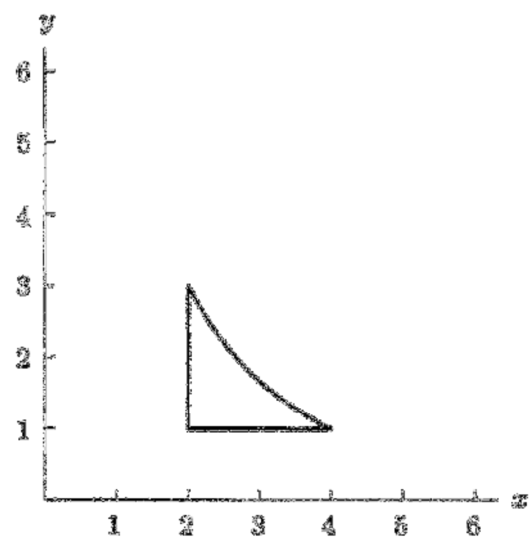
$$\sum_{n=0}^{\infty} \frac{1}{(n+2)n!} = 1 .$$

VI. Applications of Integrals

Find area, volume and surface area of regions and regions revolved about an axis.
Calculate work
Calculate fluid force

74) Imagine rotating the enclosed region in the figure about three lines separately: the *x*-axis, the *y*-axis, and the vertical line at *x* = 6. This produces three different volumes. Which of the following lists those volumes in order of smallest to largest?

(A) *x*-axis, *x* = 6, *y*-axis
(B) *x*-axis, *y*-axis, *x* = 6
(C) *y*-axis , *x* = 6, *x*-axis
(D) *y*-axis, *x*-axis, *x* = 6
(E) *x* = 6, *x*-axis, *y*-axis



75) A glass container can be modeled by revolving the graph of *y* about the *x*-axis. Write an integral expression or expressions for the volume of this container.

$$y = \begin{cases} \sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}, & 0 \le x \le 11.5 \\ 2.95, & 11.5 < x \le 15 \end{cases}$$

76) Imagine that the region between the graphs of *f* and *g* in the figure is rotated about the *y*-axis to form a solid. Circle the appropriate method to find the volume of this solid.

disks **washers** **shells**

Which of the following represents the volume of this solid?

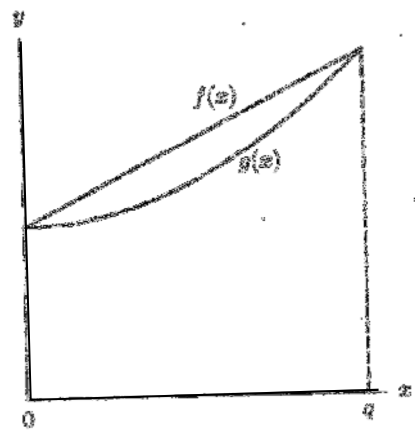
(A) $\int_0^q 2\pi x\left(f(x)-g(x)\right)dx$

(B) $\int_0^q \left(f(x)-g(x)\right)dx$

(C) $\int_0^q \pi\left(f(x)-g(x)\right)^2 dx$

(D) $\int_0^q \pi\left(\left[f(x)\right]^2-\left[g(x)\right]^2\right)dx$

(E) $\int_0^q 2\pi x\left(g(x)-f(x)\right)dx$



77) Consider the integral $\int x\tan^{-1} x dx$. Evaluate this integral using integration by parts in the usual way except choose a constant of integration to use when assigning *v* during integration by parts which will make the subsequent integration trivial.

VII. Parametric Equations

Be able to convert between Cartesian and parametric forms.
Be able to write parametric equations
Be able to calculate slopes of curves given in parametric form
Be able to calculate the length of a curve and the surface area of revolution for a curve in parametric form

78) Why are parametric equations preferred over Cartesian equations when describing the path of an object in motion?

VIII. Polar Equations

Be able to convert between polar and Cartesian equations
Be able to find the slope of a curve given in polar form
Be able to integrate polar equations in order to find area
Be able to calculate the length of a curve and the surface area of revolution for a curve in polar form

79) Explain the difference geometrically in finding area in Cartesian coordinates versus polar coordinates.

Answers to selected problems

1)

$$\int_0^1 \frac{1}{\sqrt{a}+1}\,da == 2-2\ln 2$$ (U-Substitution)

2)

$$\int \sin^2\alpha\cos^2\alpha\ d\alpha = \frac{1}{8}\left(\alpha-\frac{1}{4}\sin 4\alpha\right)+C$$ (Trigonometric Identities)

3)

$$\int_{\frac{3\pi}{4}}^{\pi} \sqrt{1+\tan^2\beta}\ d\beta == \ln\left(\sqrt{2}+1\right)$$ (Trigonometric Identities)

4)

$$\int \frac{b^2}{b^2+1}\,db = b-\tan^{-1}b+C$$ (Polynomial Division)

5)

$$\int \frac{c}{\sqrt{c^2+16}}\,dc = \sqrt{c^2+16}+k$$ (U-Substitution)

6)

$$\int \cot^2\delta\ d\delta = -\cot\delta-\delta+C$$ (Trigonometric Identities)

7)

$$\int \cos^3\phi\ d\phi = \sin\phi-\frac{1}{3}\sin^3\phi+C$$ (Trigonometric Identities and U-Substitution)

8)

$$\int g^3e^{-g}dg = -e^{-g}\left(g^3+3g^2+6g+6\right)+C$$ (Integration by Parts via a Table)

9)

$$\int_0^1 \frac{e^h}{1+e^{2h}}\,dh = \tan^{-1} e - \frac{\pi}{4}$$ (U-Substitution)

10)

$$\int \frac{k+17}{k^2+4k-5}\,dk = -2\ln|k+5| + 3\ln|k-1| + C$$ (Partial Fractions)

11)

$$\int \cos 2\phi \cos 4\phi \; d\phi = \frac{1}{2}\left(-\frac{1}{2}\sin(-2\phi) + \frac{1}{6}\sin(6\phi)\right) + C$$ (Trigonometric Identities)

12)

$$\int \sec^3 \gamma \; d\gamma = \frac{1}{2}\left(\sec\gamma \tan\gamma + \ln|\sec\gamma + \tan\gamma|\right) + C$$ (Integration by Parts and Trig. Identity)

13)

$$\int \sec(7\lambda) d\lambda = \frac{1}{7}\ln|\sec 7\lambda + \tan 7\lambda| + C$$

14)

$$\int e^{2n} \sin 3n \; dn = \frac{2}{13}e^{2n}\sin 3n - \frac{3}{13}e^{2n}\cos 3n + C$$ (Integration by Parts Twice)

15)

$$\int \frac{p^2+2}{3p^2}\,dp = \frac{1}{3}p - \frac{2}{3p} + C$$ (Separate Fractions)

16)

$$\int \sec^4 \mu \; d\mu = \tan\mu + \frac{1}{3}\tan^3 \mu + C$$ (Trigonometric Identities and U-Substitution)

17)

$$\int q2^{q^2}\,dq = \frac{1}{2\ln 2}2^{q^2} + C$$ (U-Substitution)

18)

$$\int_1^{3/2} \frac{1}{\sqrt{2r-r^2}}\,dr = \left[\sin^{-1}(r-1)\right]_1^{3/2} = \frac{\pi}{6}$$ (Completing the Square)

19)

$$\int \frac{1+s}{1+s^2}\,ds = \tan^{-1}(s) + \frac{1}{2}\ln(1+s^2) + C$$ (Separating fractions, U-Substitution)

20)

$$\int \frac{\sqrt{9-t^2}}{t^2}\,dt = -\frac{\sqrt{9-t^2}}{t} - \sin^{-1}\left(\frac{t}{3}\right) + C$$ (Trigonometric Substitution)

21)

$$\int \tan^3 \nu\,d\nu = \frac{1}{2}\tan^2\nu + \ln\left|\cos\nu\right| + C \text{ or } \frac{1}{2}\tan^2\nu - \ln\left|\sec\nu\right| + C$$ (Powers of Tangent and Secant)

22)

$$\int_{\pi/2}^{3\pi/2} \sqrt{1-\cos\theta}\,d\theta = \sqrt{2}\int_{\pi/2}^{3\pi/2} \sin\left(\frac{\theta}{2}\right)d\theta = \left[-2\sqrt{2}\cos u\right]_{\pi/4}^{3\pi/4} = 4$$, where $u = \frac{\theta}{2}$

(Trig. Identity, U-Substitution)

23)

$$\int \frac{1+\cos\rho}{\sin\rho}\,d\rho = \ln\left|1-\cos\rho\right| + C$$

(Separating Fractions)

24)

$$\int \frac{\ln v}{v}\,dv = \frac{1}{2}\ln^2 v + C$$

(U-Substitution)

25)

$$\int \sin^3\tau\cos^7\tau\ d\tau = \frac{1}{10}\cos^{10}\tau - \frac{1}{8}\cos^8\tau + C$$

(Products of Powers of Sine and Cosine)

26)

$$\int \frac{w^2\,dw}{\left(w^2-1\right)^{5/2}} = -\frac{w^3}{3\left(w^2-1\right)^{3/2}} + C,\ w>1$$

(Trigonometric Substitution, Products of Powers of Cosecant and Cotangent)

27)

$$\int \frac{1}{\csc\omega+\cot\omega}\,d\omega = -\ln\left|\csc\omega+\cot\omega\right| - \ln\left|\sin\omega\right| + C$$

(Multiplying by a Form of One)

28)

$$\int \frac{1}{x^2-4x+9}\,dx = \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x-2}{\sqrt{5}}\right) + C$$

(Completing the Square, U-Substitution)

29)

$$\int \frac{5y-9}{y^2-9}\,dy = \ln\left|y-3\right| + 4\ln\left|y+3\right| + C$$

(Partial Fractions)

30)

$$\int \frac{\sin\psi}{\sqrt{\cos\psi}}\,d\psi = -2\sqrt{\cos\psi} + C$$ (U-Substitution)

31)

$$\int \frac{z^2}{z-1}\,dz = \frac{1}{2}z^2 + z + \ln|z-1| + C$$ (Improper Fraction)

32)

$$\int_0^{\pi/3} \tan(\theta)\ln(\cos(\theta))\,d\theta = \left[-\frac{\ln^2\cos\theta}{2}\right]_0^{\pi/3} = -\frac{1}{2}\ln^2 2$$ (Integration by Parts)

33)

$$\int t^6\cos 2t\,dt = \frac{1}{2}t^6\sin 2t + \frac{3}{2}t^5\cos 2t - \frac{15}{4}t^4\sin 2t - \frac{15}{2}t^3\cos 2t + \frac{45}{4}t^2\sin 2t + \frac{45}{4}t\cos 2t - \frac{45}{8}\sin 2t + C$$

(Integration by Parts)

34)

$$\int \frac{7}{\left(m^2+9\right)^3}dm = \frac{7}{648}\tan^{-1}\left(\frac{m}{3}\right) + \frac{7m}{216(m^2+9)} + \frac{7m}{36(m^2+9)^2} + C$$ (Trigonometric substitution,

Trig. Identities)

46) Divergent , LCT
47) Absolutely Convergent, ACT/Ratio
48) Convergent, Ratio
49) Divergent, Geometric
50) Divergent , DCT
51) Divergent , By Definition
52) Divergent, Nth Term Test
53) Convergent, Ratio
54) Divergent, Nth Term Test
55) Conditionally Convergent , AST/LCT
56) Convergent , DCT
57) Convergent , LCT

58) Divergent, Sum of Conv. and Div. Geometric

59) Convergent, Ratio

60) Converges to $\frac{1}{2}$, Telescoping Absolutely Convergent, Comparison and AST or ACT

61) Converges to $\frac{1}{e-1}$, Geometric

62) Divergent, LCT

63) Converges to $\frac{67}{147}$, two geometric series

64) Convergent, Comparison Test

65) Convergent , Root Test

66) Divergent, Nth Term Test

67) Divergent, Integral Testt

68) Divergent, Ratio Test

69) Conditionally Convergent, AST/Integral Test or Comparison

70) Convergent, LCT or Comparison

71) Divergent, Nth Term Test