Name: Key

Unit One Exam

MATH 242-024

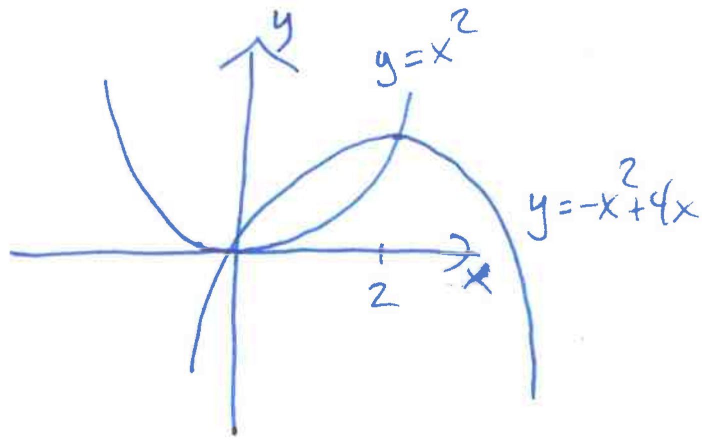
Calculus II

Instructor: Grondahl

Show All Work

Justify All Conclusions

1) Find the area enclosed by $y = x^2$ and $y = -x^2 + 4x$



$$x^2 = -x^2 + 4x$$

$$2x^2 - 4x = 0$$

$$2x(x-2) = 0$$

$$x = 0, 2$$

$$A = \int_0^2 (-x^2 + 4x - x^2)\,dx$$

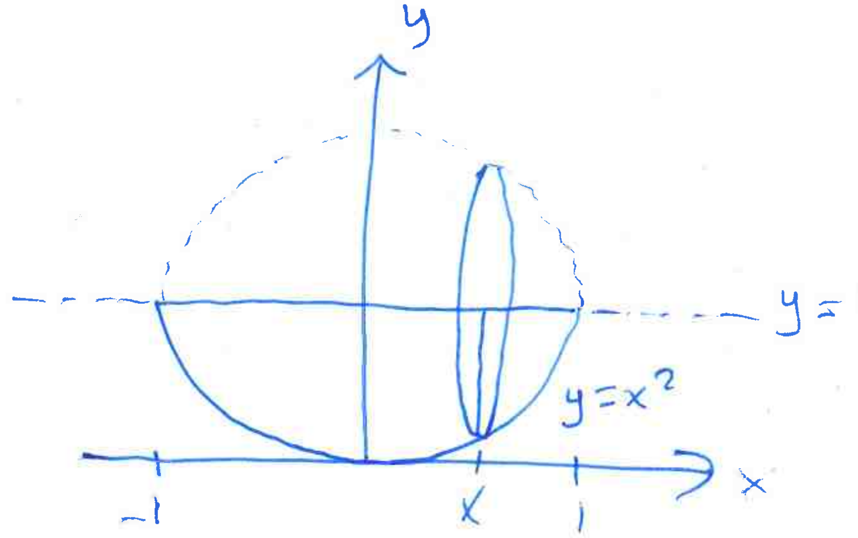
$$= \int_0^2 (-2x^2 + 4x)\,dx$$

$$= \left[-\frac{2}{3}x^3 + 2x^2\right]_0^2$$

$$= \frac{-16}{3} + 8$$

$$= \frac{8}{3}$$

2) Find the volume of the solid generated in revolving the region enclosed by $y = x^2$ and $y = 1$ about the line $y = 1$



$$A(x) = \pi r^2 = \pi(1-x^2)^2$$

$$V = \int_{-1}^{1} \pi(1-x^2)^2 dx$$

$$= \pi \int_{-1}^{1} (1-2x^2+x^4) dx$$

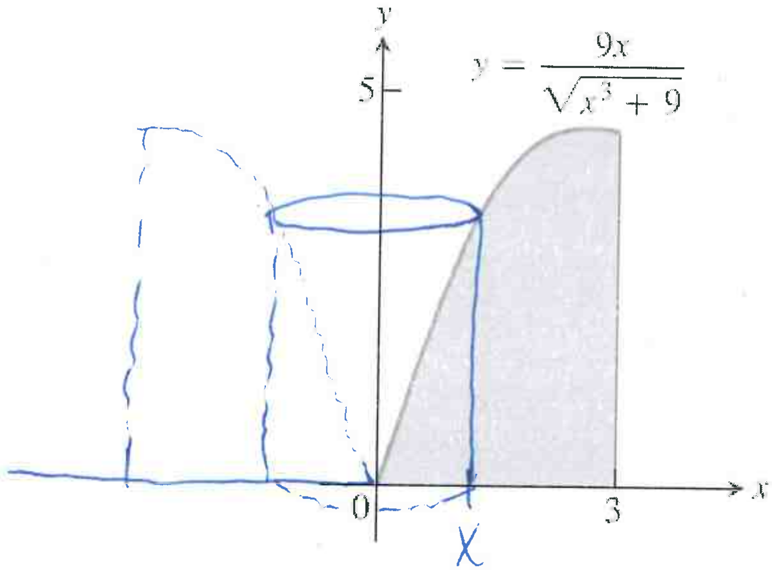
$$= \pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_{-1}^{1}$$

$$= \pi \left[(1 - 2/3 + 1/5) - (-1 + 2/3 - 1/5)\right]$$

$$= \pi (2 - 4/3 + 2/5)$$

$$= \frac{16\pi}{15}$$

3) Find the volume generated in revolving the region below about the y-axis.



$$\boxed{A(x) = \frac{18\pi x^2}{\sqrt{x^3+9}}} \quad h = \frac{9x}{\sqrt{x^3+9}}$$

$C = 2\pi r = 2\pi x$

$$V = \int_0^3 \frac{18\pi x^2}{\sqrt{x^3+9}}\,dx \qquad \begin{array}{l} u = x^3+9 \\ du = 3x^2\,dx \end{array}$$

$$= \int_9^{36} \frac{6\pi}{\sqrt{u}}\,du$$

$$= 12\pi\left[\sqrt{u}\right]_9^{36}$$

$$= 12\pi(6-3)$$

$$= 36\pi$$

4) Find the length of $x = \int_0^y \sqrt{\sec^4 t - 1}\, dt$, $\frac{-\pi}{4} \le y \le \frac{\pi}{4}$

$$\frac{dx}{dy} = \sqrt{\sec^4 y - 1}$$

$$\left(\frac{dx}{dy}\right)^2 = \sec^4 y - 1$$

$$L = \int_{-\pi/4}^{\pi/4} \sqrt{1 + \sec^4 y - 1}\; dy$$

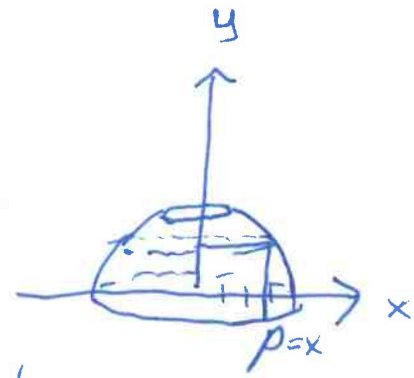
$$= \int_{-\pi/4}^{\pi/4} \sec^2 y \; dy$$

$$= \Big[\tan y\Big]_{-\pi/4}^{\pi/4}$$

$$= 1 + 1$$

$$= 2$$

5) Find the surface area generated in revolving $x = 2\sqrt{4-y}$, $0 \le y \le \frac{15}{4}$ about the y-axis



$$S.A. = \int_a^b 2\pi p \, ds$$

$$\frac{dx}{dy} = 2\left(\frac{1}{2}\right)(4-y)^{-1/2}(-1) = \frac{-1}{\sqrt{4-y}}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4-y}$$

$$S.A. = \int_0^{15/4} 2\pi\left(2\sqrt{4-y}\right)\sqrt{1+\frac{1}{4-y}} \, dy$$

$$= 4\pi \int_0^{15/4} \sqrt{5-y} \, dy$$

$u = 5-y$

$du = -dy$

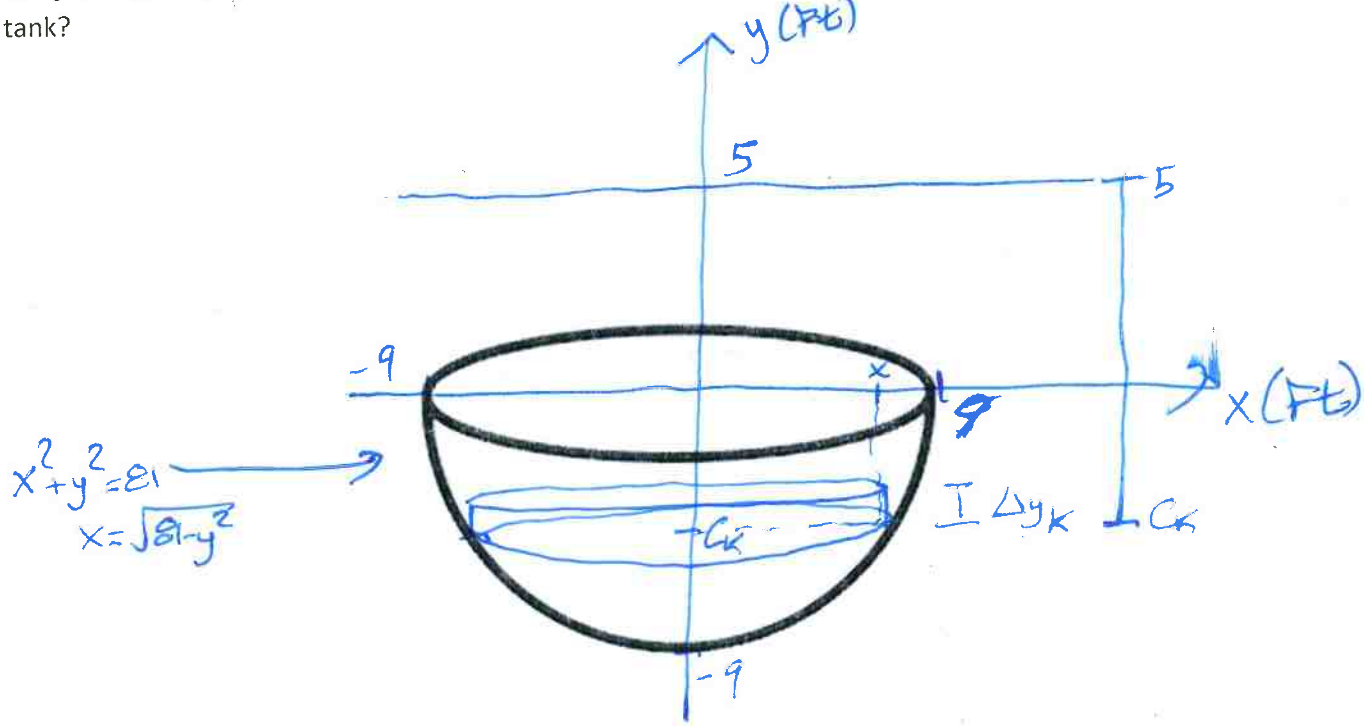
$$= -4\pi \int_5^{5/4} \sqrt{u} \, du$$

$$= \frac{-8\pi}{3}\left[u^{3/2}\right]_5^{5/4}$$

$$= \frac{-8\pi}{3}\left(\frac{5\sqrt{5}}{8} - 5\sqrt{5}\right)$$

$$= \frac{35\pi\sqrt{5}}{3}$$

6) A hemispherical tank of radius 9 is filled completely with water of weight-density $62.4\,\mathrm{lb/ft^3}$. Set up the integral that would find how much work is done on the water in pumping it to a level 5 ft. above the top of the tank?



a. Set up the Integral. Do not attempt to evaluate the integral.

$$V_K = \pi r^2 h = \pi x^2 \Delta y_K = \pi\left(\sqrt{81-c_K^2}\right)^2 \Delta y_K$$

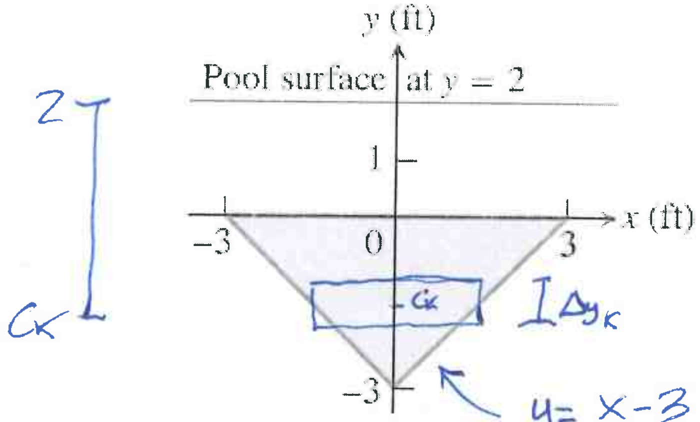
$$W_K = wV_K d = 62.4\pi\left(81-c_K^2\right)\Delta y_K\,(5-c_K)$$

$$W = \int_{-9}^{0} 62.4\pi\left(81-y^2\right)(5-y)\,dy$$

b. Give the units the answer would have

Ft-lb

7) Assuming a weight density of 50 $\mathrm{lb/ft^3}$ find the fluid force on one side of the plate shown in the diagram.



$y = x - 3$
$x = y + 3$

a. Set up the integral, **but do not solve**

$$F_k = whA$$
$$= (50)(2-c_k)(2)(c_k+3)\Delta y_k$$

$$\int_{-3}^{0} 100(2-y)(y+3)\,dy$$

b. Give the units that the answer should have

lb

8) It takes 6N of force to stretch a spring from its natural length of 1m to a length of 1.5m.

a. Set up the integral that would calculate the work done in stretching this spring from a length of 2m to a length of 5m?

(Set Up Only. Do Not Solve)

$F = kx$

$6 = k(1/2)$

$12 = k$

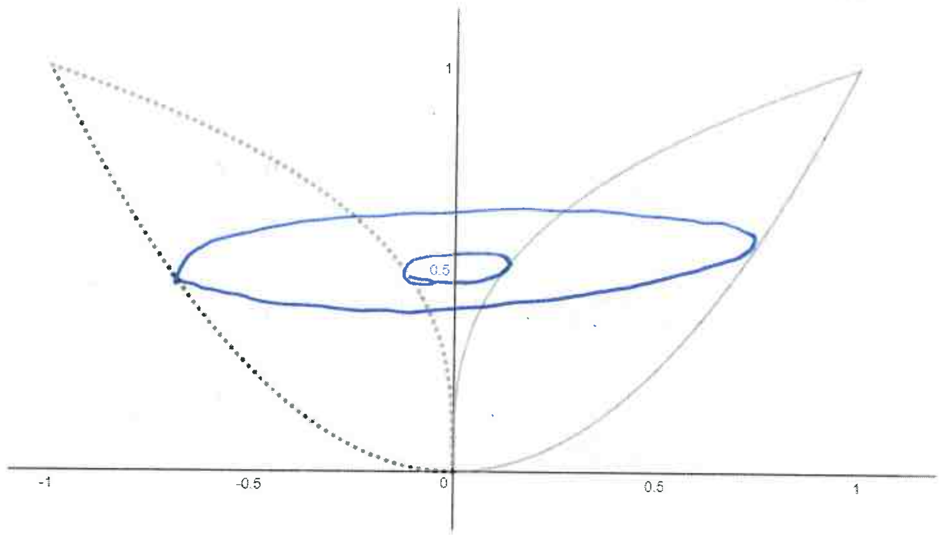
$$W = \int_1^4 12x \, dx$$

b. Give the units your answer would have

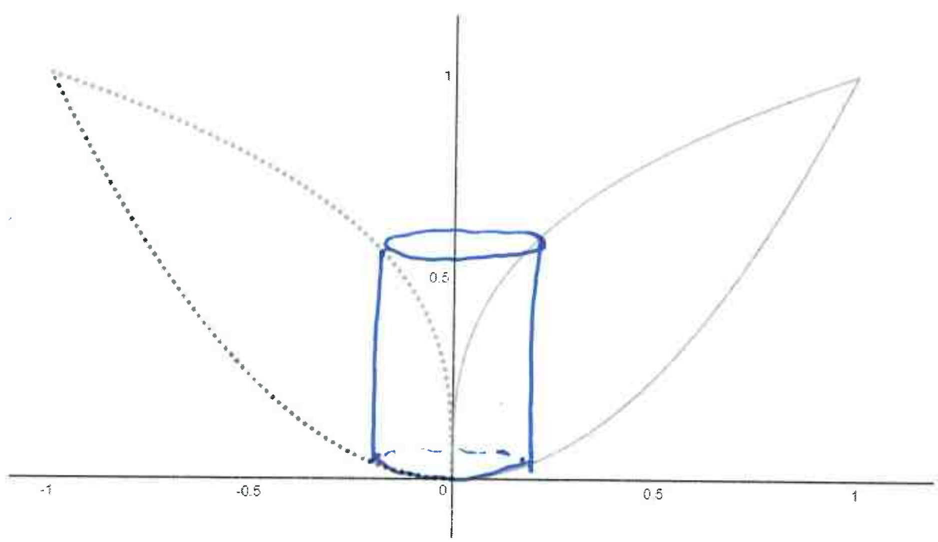
J or Nm

9) On the graphs below, sketch an arbitrary slice by cross-sections and an arbitrary slice by cylindrical shells in revolving about the y-axis.

Cross-Section



Cylindrical Shell



10) In physics, pressure is weight-density times depth. Why do we need to use integration when finding total fluid force on a vertical surface?

ON A VERTICAL SURFACE THE PRESSURE VALUES WITH DEPTH REQUIRING INTEGRATION TO SUM THE TOTAL FLUID FORCE.